Characterization of flow through porous media

UNIVERSITY OF ROME TOR VERGATA

BACHELOR DEGREE IN ENGINEERING SCIENCES

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Motivation: why oil?

Introduction

Fluid motion through porous media is governed by the conservation of:

- Mass
- Momentum
- Energy

Darcy's Law (1856)

The tortuous structure of porous media naturally contributes to complicated fluid transport through the pores.

Let's restrict the theory to the relatively simple cases of isothermal, slightly compressible single-phase fluid

Properties of rocks and fluids

ROCKS

Porosity

$$\phi = \frac{V_p}{V_b}$$

Compressibility

$$c_r = \left(\frac{1}{\phi}\frac{\partial \phi}{\partial p}\right)_T$$

Permeability

$$k = \frac{QL}{Ah}$$

$$k = \frac{2.3aL}{At}\log\left(\frac{h_1}{h_2}\right)$$

FLUIDS

Viscosity

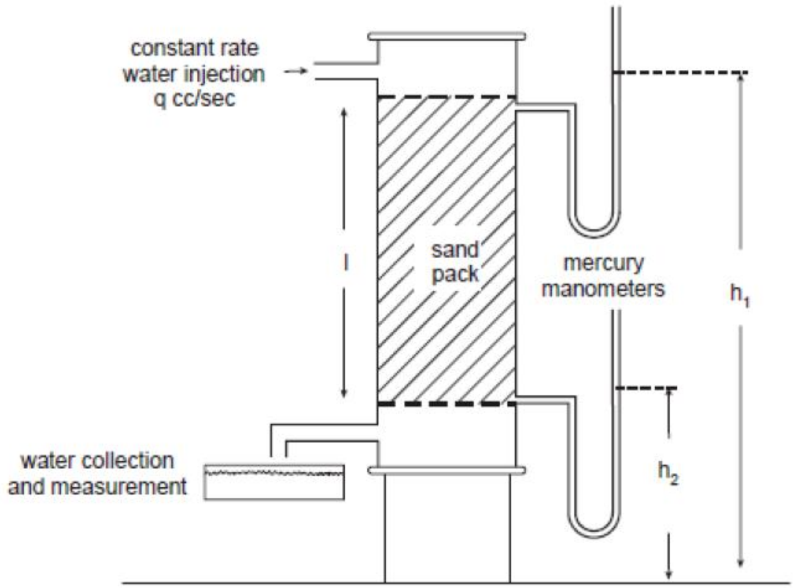
$$\mu = \frac{\tau}{\frac{\partial u}{\partial y}}$$

Compressibility

$$c_f = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

Governing equations of single-phase flow

Darcy’s Law



Conservation of momentum in flow through porous media is usually expressed with Darcy's law, an experimental relationship showed on the basis of several experiments on water filtration through sand beds

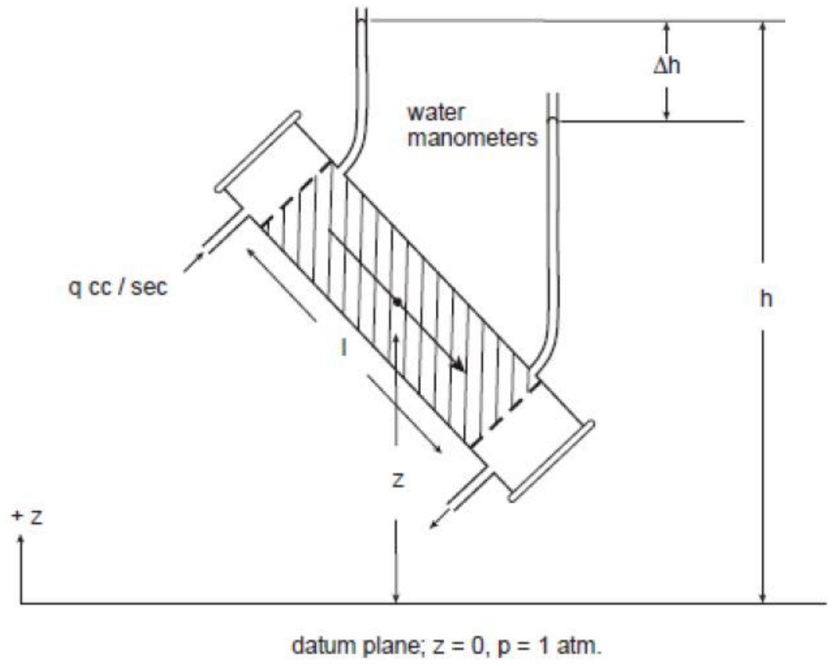
$$v = k\frac{h_1 - h_2}{l} = k\frac{\Delta h}{l}$$

Which in differential form is

$$v = k\frac{dh}{dl}$$

Governing equations of single-phase flow

Darcy’s law



Irrespective of the orientation of the sand pack, the difference in height, Δh, was always the same for a given flow rate. Thus Darcy's experimental law proved to be independent of the direction of flow in the earth's gravitational field

The pressure at any point in the flow path, which has an elevation z, relative to the datum plane, can be expressed as:

$$p = \rho g \left(h - z\right)$$

which can alternatively be expressed as:

$$\left(\frac{p}{\rho} + gz\right) = hg$$

Governing equations of single-phase flow

Darcy's law

$$\left(\frac{p}{\rho} + gz\right) = hg \Rightarrow v = \frac{k}{g}\frac{d}{dl}\left(\frac{p}{\rho} + gz\right)$$

where both terms have the same units: distance x force per unit mass

fluid potential energy per unit mass $\Phi = \frac{p}{\rho} + gz$

hence the final expression of the Darcy's Law is

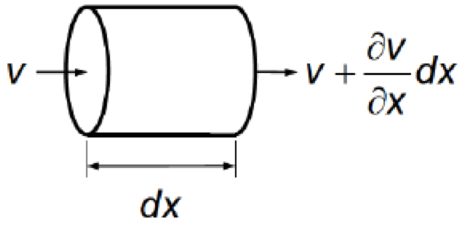
$$v = \frac{k\rho}{\mu}\frac{d\Phi}{dl} = \frac{k}{\mu}\frac{dp}{dl}$$

Governing equations of single-phase flow

Conservation of Mass

Let’s consider one-dimensional, horizontal, isothermal single-phase flow through a porous medium with constant cross-sectional area A. The conservation of mass will be:

$$A\rho v - A\left(\rho + \frac{\partial p}{\partial x}dx\right)\left(v + \frac{\partial v}{\partial x}dx\right) - A\frac{\partial \rho\phi}{\partial t}dx = 0$$



$$\frac{\partial \rho v}{\partial x} + \frac{\partial \rho\phi}{\partial t} = 0$$

Governing equations of single-phase flow

Flow equation

Substituting the Darcy's equation into the continuity equation:

$$\frac{\partial}{\partial x}\left(\rho \frac{k}{\mu}\frac{\partial p}{\partial x}\right) = \frac{\partial(\rho\phi)}{\partial t} \quad \Rightarrow \quad \frac{\partial^2 p}{\partial x^2} = \frac{\phi c \mu}{k}\frac{\partial p}{\partial t}$$

In order to solve the equation, we need to specify one initial condition and two boundary conditions

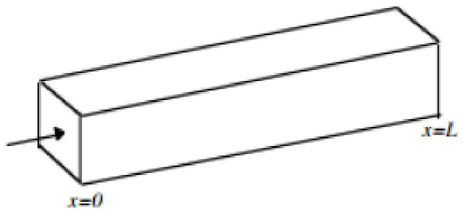
Initial Condition

$$p(x, t=0) = p_i$$

Dirichelet's BC's

$$p(x=0, t) = p_{inlet}$$

$$p(x=L, t) = p_{outlet}$$



Neumann's BC's

$$q_{inlet} = -\frac{kA}{\mu}\left(\frac{\partial p}{\partial x}\right)_{x=0}$$

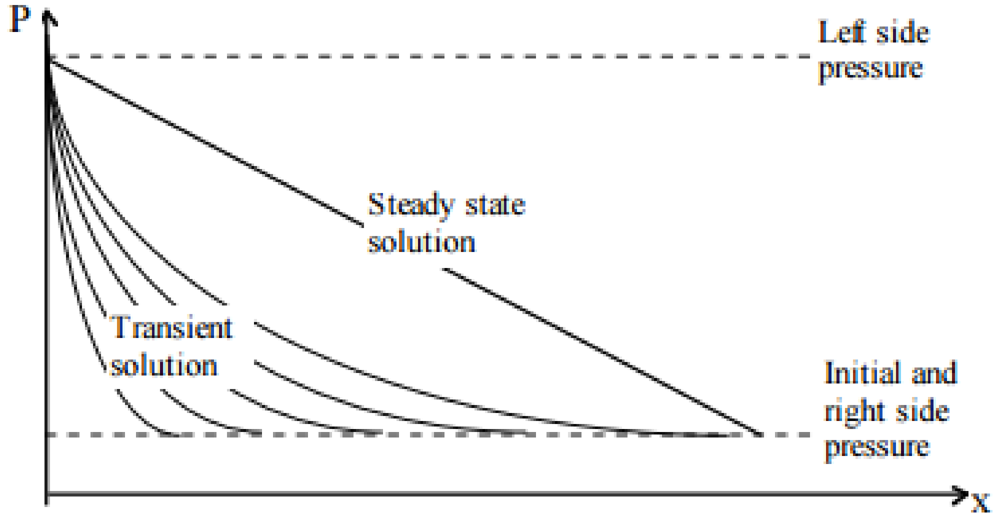
$$q_{outlet} = -\frac{kA}{\mu}\left(\frac{\partial p}{\partial x}\right)_{x=L}$$

Governing equations of single-phase flow

Flow equation

Using the aforesaid set of conditions, the following solution may be obtained:

$$p(x,t) = p_{inlet} + (\Delta p)\left[\frac{x}{L} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2}{L^2}\frac{k}{\phi c\mu}t\right)\sin\left(\frac{\phi n x}{L}\right)\right]$$



Groundwater and Petroleum

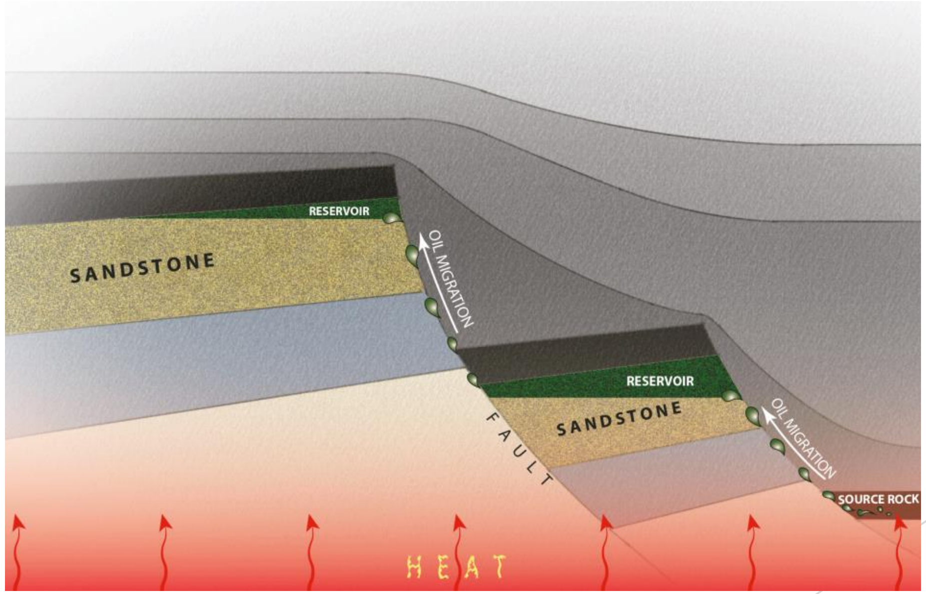
Primary Migration

water and entrained petroleum are expelled from the fine-grained source sediments into the more permeable aquifers of a sedimentary system.

$$\sigma_T = \sigma_e + p$$

Secondary Migration

movement of petroleum and water through the aquifer system to the structural and stratigraphic traps, where oil and gas pools are formed.



Conclusions

The understanding of fluid flowing through porous media allowed a strong development in several engineering fields, such as petroleum engineering.

In the last decades, various techniques have been implemented for increasing the amount of crude oil that can be extracted from a reservoir.

Several enhanced recovery techniques have been used in order to maximize the oil recovery factor

Thank you for your attention!